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# Brane-world and holography

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Abstract: We consider the brane-world in the holographic point of view. Bearing the realistic models in mind, the bulk massless scalar field is introduced. First of all, we find the constraint on the coupling of the scalar fields with the matter (not holographic CFT) on the brane. We show that the traceless part of the energy-momentum tensor of holographic CFT is a part of the bulk Weyl tensor. The trace part which comes from the trace-anomaly is corresponding to the $\rho^2$-term appeared in the generalized FRW equation in the brane-world.

Keywords: AdS-CFT Correspondence, Classical Theories of Gravity.



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## Contents





## 1. Introduction

The Randall-Sundrum brane-world is the attractive and simple model which describes the stringy cosmology [1, 2], where the 4-dimensional spacetime is regarded as a 4-dimensional Minkowski spacetime embedded in the 5-dimensional anti-de Sitter (AdS) spacetime. In this model, it has been checked that the Newton gravity is recovered in the linearized theory [1]–[6], and in special cases, it has recently been confirmed that the Einstein gravity is recovered up to the second order perturbation analysis [7]. Due to the technical reason, however, the non-linear aspects of the brane-world are still unclear (See ref. [8] for some attempts on this issue). It is remarkable that the homogeneous-isotropic universe is described by the domain wall motion in the 5-dimensional Schwarzschild-AdS spacetime [9, 10].

We shall consider in this paper that the gravitational equations on the brane from the holographic point of view. We will compare the gravitational equations, which are obtained via the purely geometrical reduction without any approximations, with the effective equations derived by using the AdS/CFT correspondence [11]. This direction was initiated by Witten, and then formulated by others [6, 12, 13, 14]. They have shown that the correction to the Newton gravity can be calculated by CFT living on the brane(See ref. [15] for the cosmological applications). In connection with this topic, other aspects of the dilatonic brane-world have been actively investigated so far [16]–[21].

In this paper we will consider what kind of brane-world is compatible to the AdS/CFT correspondence. There are two ways to obtain the effective equation on the brane. By comparing them, our purpose should be attained. The rest of this report is organised as follows. In section 2, following refs. [5, 18], we give a brief review of the gravitational equations on the brane. In section 3, we derive the effective equations on the brane via the AdS/CFT correspondence and compare them with those described in section 2. To make the points clear, we consider a simple model there. The model, which can be described by the aspect of the AdS/CFT correspondence, has a certain constraint on the coupling of the scalar field to the matters on the brane. Finally, we give a summary in section 4.

## 2. The gravitational equation on the brane

In this section, we briefly review the gravitational equation on the brane derived in refs. [5, 18]. We begin with the action

$$
\begin{aligned}
S &= S_{\rm bulk} + S_{\rm brane} \\
&= \left\{ \int d^5x\sqrt{-g}\left[\frac{1}{2{\kappa_5}^2}{}^{(5)}R - \frac{1}{2}(\nabla\phi)^2 - V(\phi)\right] + \int d^4x\sqrt{-g}\frac{1}{{\kappa_5}^2}K \right\} + \\
&\quad + \int d^4x\sqrt{-q}\Big[\mathcal{L}_{\rm m} - \lambda(\phi)\Big] .
\end{aligned}
\tag{2.1}
$$

We here work on the gaussian normal coordinate such that the metric takes the form

$$
ds^2 = g_{MN}dx^M dx^N = dy^2 + q_{\mu\nu}(y,x)dx^\mu dx^\nu . \tag{2.2}
$$

The brane is assumed to locate at $y = 0$ and $q_{\mu\nu}(0,x) =: q_{\mu\nu}(x)$ is the induced metric on the brane. From the Gauss equation and the Israel's junction condition [22], the gravitational equation on the brane is given by

$$
{}^{(4)}G_{\mu\nu} = \frac{2{\kappa_5}^2}{3}\hat{T}_{\mu\nu}(\phi) + g_{\mu\nu}\left[-\Lambda_4 + \frac{{\kappa_5}^2}{16}\left(2\frac{d\lambda}{d\phi} + \frac{\partial\mathcal{L}_m}{\partial\phi}\right)\frac{\partial\mathcal{L}_m}{\partial\phi}\right] + 8\pi G_4(\phi)\tau_{\mu\nu} + {\kappa_5}^4\pi_{\mu\nu} - E_{\mu\nu} , \tag{2.3}
$$

where

$$
\begin{aligned}
\hat{T}_{\mu\nu}(\phi) &= D_\mu\phi D_\nu\phi - \frac{5}{8}q_{\mu\nu}(D\phi)^2 , \\
\Lambda_4 &= \frac{1}{2}{\kappa_5}^2\left[V + \frac{1}{6}{\kappa_5}^2\lambda^2 - \frac{1}{8}\left(\frac{d\lambda}{d\phi}\right)^2\right] , \\
8\pi G_4(\phi) &= \frac{{\kappa_5}^4}{6}\lambda(\phi) , \\
\pi_{\mu\nu} &= -\frac{1}{4}\tau_{\mu\alpha}{\tau_\nu}^\alpha + \frac{1}{12}\tau\tau_{\mu\nu} + \frac{1}{8}q_{\mu\nu}\tau_{\alpha\beta}\tau^{\alpha\beta} - \frac{1}{24}q_{\mu\nu}\tau^2 ,
\end{aligned}
\tag{2.4}
$$

and

$$
E_{\mu\nu} = {}^{(5)}C_{\mu M\nu N}n^M n^N . \tag{2.5}
$$

$\tau_{\mu\nu}$ is the energy-momentum tensor of $\mathcal{L}_m$ on the brane. $D_\mu$ is the covariant derivative with respect to $q_{\mu\nu}$. Here we have taken into account a possibility that the matter fields on the brane couple to the dilaton field $\phi$. The second-rank trace-free symmetric tensor $E_{\mu\nu}$ is the so-called "electric" part of the 5-dimensional Weyl tensor, ${}^{(5)}C_{KLMN}$. Note that we did not use any approximation to derive the gravitational equations. In this sense, eq. (2.3) is exact under the assumption that the full action is given by eq. (2.1). Since $E_{\mu\nu}$ is the 5-dimensional quantity, however, the above equations are not a closed system in the 4-dimensional sense. To evaluate $E_{\mu\nu}$ on the brane, we must solve the equation for $E_{\mu\nu}$ in the bulk [4, 23].

In a similar way, we obtain the equation for the scalar field on the brane:

$$
D^2\phi - \frac{{\kappa_5}^2}{12}(4\lambda - \tau)\left(\frac{d\lambda}{d\phi} + \frac{\partial\mathcal{L}_m}{\partial\phi}\right) - \frac{dV}{d\phi} = -\partial_y^2\phi|_{\rm brane} . \tag{2.6}
$$



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The trace part of the gravitational equation on the brane will play an important role for the discussion in the next section:

$$ {}^{(4)}R = {\kappa_5}^2 (D\phi)^2 + 4\left[\Lambda_4 - \frac{{\kappa_5}^2}{16}\left(2\frac{d\lambda}{d\phi} + \frac{\partial \mathcal{L}_m}{\partial \phi}\right)\frac{\partial \mathcal{L}_m}{\partial \phi}\right] - 8\pi G_4 \tau - {\kappa_5}^4 \pi^\mu{}_\mu \,. \tag{2.7} $$

Hereafter we set ${\kappa_5}^2 = 1$ for the brevity.

## 3. View from the holography

### 3.1 Set-up

To derive the effective equation on the brane from the holographic point of view, we follow the Giddings, Katz and Randall's argument [6], where they started with the following observation in the path integral:

$$ \begin{aligned} Z &= \int \mathcal{D}g e^{i[S_5(g) + \frac{1}{2}S_{\rm brane}(q)]} \\ &= \int \mathcal{D}q e^{\frac{i}{2}S_{\rm brane}} \int_{g|_b = q} \mathcal{D}g e^{iS_5} \\ &= \int \mathcal{D}q e^{\frac{i}{2}S_{\rm brane}} e^{iS_{\rm ct}} \left\langle e^{i\int d^4x q_{\mu\nu}T^{\mu\nu}} \right\rangle_{\rm CFT} . \end{aligned} \tag{3.1} $$

In the above a coupling of the bulk scalar to a boundary operator is tacitly included, this is, $g$ expresses the both of the metric and scalar for the brevity. $S_5$ is the gravitational action with the boundary term, $S_{\rm brane}$ is the brane action including the brane tension and the matter fields on the brane. When we move from the second to the third lines, we have used the AdS/CFT correspondence [11]; Roughly speaking, the classical gravity in the bulk is dual with CFT living on the boundary. Then, we must introduce the counter-term $S_{ct}$ to make the action finite. The counter-term is assumed to have the local form in terms of the 4-dimensional quantities, which can be easily determined by using the Hamilton-Jacobi equation [24] (See also [25]):[1]

$$ \begin{aligned} \mathcal{H} &= \frac{2}{\sqrt{-q}}\frac{\delta S}{\delta q_{\alpha\beta}}\frac{\delta S}{\delta q_{\mu\nu}}\left(q_{\alpha\mu}q_{\beta\nu} - \frac{1}{3}q_{\alpha\beta}q_{\mu\nu}\right) + \\ &\quad + \frac{1}{2}\frac{1}{\sqrt{-q}}\left(\frac{\delta S}{\delta \phi}\right)^2 - \frac{1}{2}\sqrt{-q}(D\phi)^2 + \frac{1}{2}\sqrt{-q}\,{}^{(4)}R - \sqrt{-q}V(\phi) \\ &= 0 \,. \end{aligned} \tag{3.2} $$

This will be solved by expanding on shell action $S$ with respect to the order of the derivatives, that is, we perform the derivative expansion.

[1]The Hamilton-Jacobi equation was applied into cosmology to discuss the inhomogeneities in the long wave limit [27]. The long wave approximation seems to correspond to the low energy limit.

### 3.2 An example

In this subsection, we will show an explicit relation between the "electric" part of the 5-dimensional Weyl tensor and the energy-momentum tensor of CFT living on the brane to the 2nd order of the derivative expansion. To the 4th order, we will obtain the relation between $\pi^\mu{}_\mu$ and the trace-anomaly of CFT.

First of all, we remind that the generalized AdS/CFT argument depends on the scheme in the general bulk potential except for the trivial case. Apart from the Hamilton-Jacobi formalism adopted in this report, there is another way where we take the metric expansion near the brane (i.e., boundary) [28]. To obtain the universal result, our attention will be focused on the constant potential cases.

We here consider the system which can be controlled by the counter-term

$$S_{\rm ct} = \frac{3}{\ell}\int d^4x\sqrt{-q} + \frac{\ell}{4}\int d^4x\sqrt{-q}\left({}^{(4)}R - D_\mu\phi D^\mu\phi\right) + S^{(4)} + \cdots\,, \tag{3.3}$$

where $S^{(4)}$ is the counter term including the 4th derivatives like ${}^{(4)}R^2$ and ${}^{(4)}R(D\phi)^2$. As seen soon, the system must have the trivial potential for the bulk scalar field.

From the Hamilton-Jacobi equation of the 0th order, we obtain

$$V(\phi) = -\frac{6}{\ell^2} = \text{constant}\,. \tag{3.4}$$

Using the above counter-term in eq. (3.1), we obtain the effective gravitational equation on the brane:

$${}^{(4)}G_{\mu\nu} = 8\pi G_4\tau_{\mu\nu} + D_\mu\phi D_\nu\phi - \frac{1}{2}q_{\mu\nu}(D\phi)^2 + \frac{4}{\ell}\frac{1}{\sqrt{-q}}\frac{\delta S^{(4)}}{\delta q_{\alpha\beta}}q_{\mu\alpha}q_{\nu\beta} + \frac{4}{\ell}\langle T_{\mu\nu}\rangle_{\rm CFT}\,. \tag{3.5}$$

This equation should be compared with eq. (2.3). In the above we set the net cosmological constant on the brane to be zero, which requires

$$\lambda = \frac{6}{\ell}\,. \tag{3.6}$$

Comparing eq. (2.3) with eq. (3.5), we obtain the relation between $E_{\mu\nu}$ and the energy-momentum tensor of CFT in the 2nd derivative order or $O(T_{\mu\nu})$:

$$E_{\mu\nu} = -\frac{4}{\ell}\left\langle T_{\mu\nu} - \frac{1}{4}q_{\mu\nu}T\right\rangle_{\rm CFT} - \frac{1}{3}\left[D_\mu\phi D_\nu\phi - \frac{1}{4}q_{\mu\nu}(D\phi)^2\right]. \tag{3.7}$$

Although $\langle T_{\mu\nu}\rangle_{\rm CFT}$ is expected to be higher order than $(D\phi)^2$ terms, we keep it in the right-hand side in eq. (3.7).

The trace of the effective equation is

$${}^{(4)}R = -8\pi G_4\tau + (D\phi)^2 - \frac{4}{\ell}\langle T^\mu{}_\mu\rangle_{\rm CFT}\,, \tag{3.8}$$

where we used the fact that the tensor produced from the 4th order local term $S^{(4)}$ is traceless:

$$\frac{\delta S^{(4)}}{\delta q_{\mu\nu}}q_{\mu\nu} = 0\,. \tag{3.9}$$

The trace-anomaly of CFT can also be evaluated by using the Hamilton-Jacobi equation [24, 26] and [28]–[31]:

$$\langle T^\mu{}_\mu\rangle_{\rm CFT} = \frac{\ell^3}{16}\biggl[{}^{(4)}R_{\mu\nu}{}^{(4)}R^{\mu\nu} - \frac{1}{3}{}^{(4)}R^2 - 2{}^{(4)}R^{\mu\nu}D_\mu\phi D_\nu\phi + \frac{2}{3}D_\mu\phi D^\mu\phi^{(4)}R + \\ + \frac{2}{3}(D_\mu\phi D^\mu\phi)^2\biggr]. \tag{3.10}$$

Let us compare eq. (2.7) with eq. (3.8). To do so we set $\Lambda_4 = 0$ in eq. (2.7). Since $\lambda$ is constant, eq. (2.7) becomes

$${}^{(4)}R = -8\pi G_4\tau + (D\phi)^2 - \frac{1}{4}\left(\frac{\partial\mathcal{L}_m}{\partial\phi}\right)^2 - \pi^\mu{}_\mu\,. \tag{3.11}$$

Using the Einstein equation to the 2nd order, ${}^{(4)}G_{\mu\nu} \simeq 8\pi G_4\tau_{\mu\nu} + D_\mu\phi D_\nu\phi - \frac{1}{2}g_{\mu\nu}(D\phi)^2$, together with eqs. (3.8) and (3.11) implies

$$\frac{\partial\mathcal{L}_m}{\partial\phi} = 0\,. \tag{3.12}$$

This means that the coupling of the scalar field to the matter fields on the brane is not admitted. It is reminded that the holographic CFT does couples to the scalar field.

As a summary, we have shown

$$\pi^\mu{}_\mu = \frac{4}{\ell}\langle T^\mu{}_\mu\rangle_{\rm CFT}\,. \tag{3.13}$$

It is already well-known that the term, $\pi^\mu{}_\mu$, implies the modification of the FRW models [9]:

$$\left(\frac{\dot a}{a}\right)^2 = \frac{8\pi G_4}{3}\rho + \frac{1}{36}\rho^2\,. \tag{3.14}$$

Hence we can conclude that $\rho^2$-term comes from the trace-anomaly from the holographic point of view.

## 4. Summary

We considered the generic features of the brane-world from the holographic point of view. We showed that the explicit relation between the "electric" part of the Weyl tensor and the energy-momentum tensor of CFT living on the brane. This gives us the reformulation of previous works of refs. [6, 12] in the system with the bulk scalar field. In addition, the $\rho^2$-term comes from the trace anomaly of CFT.

We stress that the brane-world models with a correct AdS/CFT interpretation will belong to a rather limited class in general. As an example, we considered the bulk massless scalar field and showed that the coupling of the scalar field with the matters on the brane cannot be admitted. This is also one of our main results. On the other hand, it is reminded that holographic CFT is coupled to the bulk scalar on the branes(See eq. (3.10)).



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## A. Derivation of the counter terms

In general the 0th order of the counter terms can be written as

$$S^{(0)} = 3\int d^4x\sqrt{-q}H(\phi)\,. \tag{A.1}$$

In this order the Hamilton-Jacobi equation is

$$\frac{2}{\sqrt{-q}}\frac{\delta S^{(0)}}{\delta q_{\alpha\beta}}\frac{\delta S^{(0)}}{\delta q_{\mu\nu}}\left(q_{\alpha\mu}q_{\beta\nu}-\frac{1}{3}q_{\alpha\beta}q_{\mu\nu}\right)+\frac{1}{2}\frac{1}{\sqrt{-q}}\left(\frac{\delta S^{(0)}}{\delta\phi}\right)^2-\sqrt{-q}V(\phi)=0\,. \tag{A.2}$$

Substituting eq. (A.1) into eq. (A.2), we obtain the relation between the potential for the bulk scalar and $H(\phi)$:

$$V(\phi)=\frac{9}{2}\left[\left(\frac{dH}{d\phi}\right)^2-\frac{4}{3}H^2(\phi)\right]. \tag{A.3}$$

Since $H(\phi)=1/\ell$ in the text, we obtain eq. (3.4). In the 2nd order the Hamilton-Jacobi equation is

$$\frac{4}{\sqrt{-q}}\frac{\delta S^{(0)}}{\delta q_{\alpha\beta}}\frac{\delta S^{(2)}}{\delta q_{\mu\nu}}\left(q_{\alpha\mu}q_{\beta\nu}-\frac{1}{3}q_{\alpha\beta}q_{\mu\nu}\right)+\frac{1}{\sqrt{-q}}\frac{\delta S^{(0)}}{\delta\phi}\frac{\delta S^{(2)}}{\delta\phi}-\frac{1}{2}\sqrt{-q}(D\phi)^2+\frac{1}{2}\sqrt{-q}^{(4)}R=0\,. \tag{A.4}$$

Substituting eq. (A.1) into the above, we obtain eq. (3.3) in the text.

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